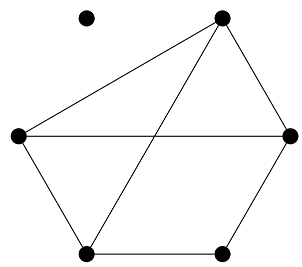
HOMEWORK 10
415G 001 COMBINATORICS AND GRAPH THEORY

DUE FRIDAY 12/2

Exercises

1. A Graph is *color critical* if the removal of any vertex decreases the graph's chromatic number. Show that every color critical graph with $\chi(G) = k$ has the following properties:
(a) G is connected.
(b) Every vertex of G has degree $\geq k - 1$.
(c) G has no vertex whose removal disconnects the graph.

2. Compute the chromatic polynomial of the following graph.



3. The *line graph* $L(G)$ of a graph G is the graph that has a vertex for every edge of G and two vertices in $L(G)$ are adjacent if the corresponding edges in G share a common end vertex. Show that G can be properly edge-colored with k colors if and only if $L(G)$ can be properly vertex-colored with k colors.

4. In a round-robin tournament where each pair of 6 contestants plays each other, a major problem is scheduling the play over a minimal number of days (each contestant plays at most one match a day). What is the minimal number of days needed for such a tournament? (Hint: Restate the problem as an edge coloring problem).

5. A graph in which every vertex degree is 3 is called a *cubic graph*.
(a) Prove that a cubic graph has to have an even number of vertices.
(b) Prove that all hamiltonian cubic graphs have chromatic index 3 (class 1).
(c) Find a non-hamiltonian cubic graph with chormatic index 4 (class 2)

Suggested exercises

From the book. 5.11, 5.12, 5.13, 5.14, 5.15, 5.16, 5.18, 5.19