HOMEWORK 2
415G 001 COMBINATORICS AND GRAPH THEORY

DUE FRIDAY 9/9

Exercises

1. What is the probability that a permutation of [6] has
(i) 1 and 2 side-by-side?
(ii) 1 occurring somewhere to the left of 2?

2. Prove the binomial theorem by mathematical induction.

3. For the identity

$$\sum_{r=0}^{k} \binom{n}{r}\binom{m}{k-r} = \binom{n+m}{k}.$$

(i) Give a proof counting a set in two different ways.
(ii) Give another proof using the binomial theorem.

4. According to Figure 1 find the number of lattice paths taking only EAST and NORTH steps such that the path travels:
(i) from A to D
(ii) from A to D and includes both the points B and C
(iii) from A to D and avoids B but includes C
(iv) from A to D and avoids both B and C
where the coordinates are $A = (0, 0)$, $B = (3, 2)$, $C = (7, 4)$ and $D = (10, 7)$.

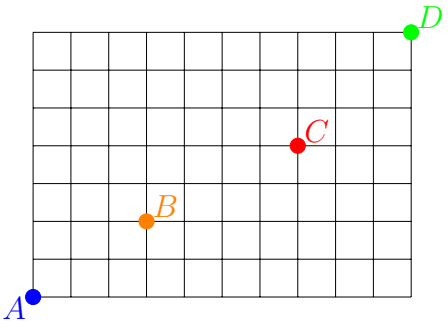

FIGURE 1

Suggested exercises. 1.1, 1.2, 1.7, 1.10, 1.17, 1.19