HOMEWORK 3
415G 001 COMBINATORICS AND GRAPH THEORY

DUE FRIDAY 9/23

Exercises

1. How many arrangements of a deck of 52 cards has exactly k runs of hearts? (A run is a group of consecutive objects)

2. How many arrangements are there with n 0's and m 1's, with k *runs* of 0's where each run of 0's has at least three 0's (so $n \geq 3k$)? (A run here is a group of consecutive letters that are repeated, for example the arrangement $\underline{0000}11\underline{000}11\underline{00000}$ has three runs of 0's and each run of 0's has at least three 0's.)

3. How many arrangements of the letters in STATISTICS have all of the following properties simultaneously?
- No consecutive Ss
- Vowels in alphabetical order
- The 3 Ts are consecutive (appear as 3 Ts in a row)

4. How many integer solutions are there to $x_1 + x_2 + x_3 + x_4 + x_5 = 31$ with
(a) $x_i \geq 0$ for all $i \in [5]$
(b) $x_i > 0$ for all $i \in [5]$
(c) $x_i \geq i$ for all $i \in [5]$

5. Prove the following theorem:

Theorem. Let M be a *multiset* with n objects, λ_1 of type 1, λ_2 of type 2, ..., and λ_k of type k, where $\lambda_1 + \lambda_2 + \cdots + \lambda_k = n$. Then the number of ordered arrangements (*multipermutations* or *multiset permutations*) of the objects in M is

$$\frac{n!}{\lambda_1!\lambda_2!\cdots\lambda_k!}.$$

(This number is also denoted $\binom{n}{\lambda_1,\lambda_2,\ldots,\lambda_k}$ and called the *multinomial coefficient*)

Suggested exercises

From the book. 1.5, 1.13, 1.14, 1.15, 1.16

Additional.

1. How many ways are there to pick a collection of 10 coins from piles of pennies, nickels, dimes and quarters?

2. How many ways are there to arrange the letters in MISSISSIPPI?

3. How many different nth-order partial derivatives does a function $f(x_1, x_2, \ldots, x_k)$ have?

4. How many ways are there to arrange the 26 letters of the alphabet so that no pair of vowels appears consecutively?