HOMEWORK 4 415G 001 COMBINATORICS AND GRAPH THEORY

DUE FRIDAY 09/30

Exercises

- 1. Find a recurrence relation for the number of rabbits after n months if
 - initially there is one pair of rabbits who were just born, and
 - every month each pair of rabbits that are more than one month old have a pair of offspring (a male and a female)
- 2. (a). Find a recurrence relation for the number R_n of regions created by n mutually overlapping circles on a piece of paper such that no three circles have a common intersection point.
 - (b). Find a closed formula for R_n .
- **3.** Find a recurrence relation to count the number of *n*-digit binary sequences with at least one instance of consecutive 0s.
- 4. Solve the recurrence

$$a_n - 5a_{n-1} + 8a_{n-2} - 4a_{n-3} = 3^n$$

with initial conditions $a_1 = 1$, $a_2 = 5$ and $a_3 = 17$.

Suggested exercises

From the book. 2.1, 2.2, 2.3, 2.5, 2.7, 2.9, 2.11, 2.14

Additional.

1. A partition of the set [n] is a disjoint collection $\{B_1, B_2, \ldots, B_k\}$ of subsets $B_i \subseteq [n]$ (called the *blocks* of the partition) such that $[n] = \bigcup_{i=1}^k B_i$ (the union of all the blocks is [n]). Let S(n,k) be the number of partitions of [n] in exactly k blocks. Find a recurrence relation for S(n,k) (Hint: the recurrence is in terms of S(n-1,k-1) and S(n-1,k)).