# HOMEWORK 4 415G 001 COMBINATORICS AND GRAPH THEORY 

DUE FRIDAY 09/30

## Exercises

1. Find a recurrence relation for the number of rabbits after $n$ months if

- initially there is one pair of rabbits who were just born, and
- every month each pair of rabbits that are more than one month old have a pair of offspring (a male and a female)

2. (a). Find a recurrence relation for the number $R_{n}$ of regions created by $n$ mutually overlapping circles on a piece of paper such that no three circles have a common intersection point.
(b). Find a closed formula for $R_{n}$.
3. Find a recurrence relation to count the number of $n$-digit binary sequences with at least one instance of consecutive 0s.
4. Solve the recurrence

$$
a_{n}-5 a_{n-1}+8 a_{n-2}-4 a_{n-3}=3^{n}
$$

with initial conditions $a_{1}=1, a_{2}=5$ and $a_{3}=17$.

## Suggested exercises

From the book. 2.1, 2.2, 2.3, 2.5, 2.7, 2.9, 2.11, 2.14

## Additional.

1. A partition of the set $[n]$ is a disjoint collection $\left\{B_{1}, B_{2}, \ldots, B_{k}\right\}$ of subsets $B_{i} \subseteq[n]$ (called the blocks of the partition) such that $[n]=\bigcup_{i=1}^{k} B_{i}$ (the union of all the blocks is $[n])$. Let $S(n, k)$ be the number of partitions of $[n]$ in exactly $k$ blocks. Find a recurrence relation for $S(n, k)$ (Hint: the recurrence is in terms of $S(n-1, k-1)$ and $S(n-1, k)$ ).
