# HOMEWORK 5 415G 001 COMBINATORICS AND GRAPH THEORY 

DUE MONDAY 10/10

## Exercises

1. Find the ordinary generating function of the sequence $\left\{a_{n}\right\}_{n \geq 0}$ satisfying the recurrence $a_{n+2}=2 a_{n+1}+3 a_{n}$ for $n \geq 0$ with initial conditions $a_{0}=0$ and $a_{1}=1$.
2. Find the ordinary generating function for the Fibonacci sequence defined by the recursion,

$$
F_{n}=F_{n-1}+F_{n-2} \quad n \geq 2 \quad F_{0}=1, F_{1}=1
$$

and use it to find a closed formula for $F_{n}$.
3. Build a generating function for the number $a_{n}$ of ways that we can choose $2 \Phi, 3 \Phi$ and $5 థ$ stamps adding to a net value of $n \mathrm{q}$.
4. Find the following coefficients:
(a) $\left[x^{k}\right] \frac{1+a x}{1-b x}$.
(b) $\left[x^{17}\right]\left(\frac{1-x^{10}}{1-x}\right)^{6}$
(c) $\left[x^{32}\right]\left(x^{3}+x^{4}+x^{5}+x^{6}+x^{7}\right)^{7}$

## Suggested exercises

From the book. 2.4, 2.16, 2.17, 2.18, 2.20

## Additional.

1. How many positive integers less than $1,000,000$ have the sum of their digits equal to 17 ?
(Hint: Use generating functions)
2. Build a generating function for the number $a_{n}$ of solutions of the equation

$$
x_{1}+x_{2}+x_{3}+x_{4}=n, 2 \leq x_{i} \leq 8, x_{1} \text { odd, } x_{2} \text { even. }
$$

