HOMEWORK 5 415G 001 COMBINATORICS AND GRAPH THEORY

DUE MONDAY 10/10

Exercises

- **1.** Find the ordinary generating function of the sequence $\{a_n\}_{n\geq 0}$ satisfying the recurrence $a_{n+2} = 2a_{n+1} + 3a_n$ for $n \geq 0$ with initial conditions $a_0 = 0$ and $a_1 = 1$.
- 2. Find the ordinary generating function for the Fibonacci sequence defined by the recursion,

$$F_n = F_{n-1} + F_{n-2}$$
 $n \ge 2$ $F_0 = 1, F_1 = 1$

and use it to find a closed formula for F_n .

- **3.** Build a generating function for the number a_n of ways that we can choose 2¢, 3¢ and 5¢ stamps adding to a net value of n¢.
- 4. Find the following coefficients:

(a)
$$[x^k] \frac{1+ax}{1-bx}$$
.
(b) $[x^{17}] \left(\frac{1-x^{10}}{1-x}\right)^6$
(c) $[x^{32}](x^3+x^4+x^5+x^6+x^7)^7$

Suggested exercises

From the book. 2.4, 2.16, 2.17, 2.18, 2.20

Additional.

- 1. How many positive integers less than 1,000,000 have the sum of their digits equal to 17? (Hint: Use generating functions)
- **2.** Build a generating function for the number a_n of solutions of the equation

 $x_1 + x_2 + x_3 + x_4 = n, 2 \le x_i \le 8, x_1 \text{ odd}, x_2 \text{ even.}$