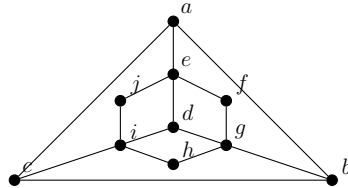# HOMEWORK 7
415G 001 COMBINATORICS AND GRAPH THEORY

DUE FRIDAY 10/28

Exercises

1. Is the following graph bipartite? Explain why.

2. Prove that if a circuit in a planar graph G encloses exactly two regions, each of which has an even number of boundary edges, then the circuit has even length. Then use this to prove that if a circuit in a planar graph G encloses a collection of regions, each of which has an even number of boundary edges, then the circuit has even length.

3. (a) Show that any planar graph can be drawn on the surface of a sphere without crossing edges and vice versa.

(b) Show that $K_{3,3}$ and K_5 can be drawn on the surface of a doughnut (torus) without crossing edges.

4. Suppose a planar graph is not connected but instead consists of several components. Find the appropriate modification of Euler's formula (and prove that your formula holds) for a planar graph with c components.

Suggested exercises

Additional.

1. If a planar graph with n vertices all of degree 4 have 10 regions, determine n.

From the book. 3.7, 3.8, 3.9, 3.10, 3.11, 3.12, 3.16