MA 714 TOPICS IN DISCRETE MATHEMATICS: POSET COHOMOLOGY AND COMBINATORIAL SPECIES

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Course Description: The notion of order is a fundamental idea that is pervasive across almost every subject in mathematics. The formal study of partially ordered sets (posets) can be traced back to the nineteenth century but it had its major development in the twentieth century after the work of Birkhoff, among others, in lattice theory; Weisner, Hall and Ward in the theory of Möbius functions; and the seminal work of Gian-Carlo Rota in 1964 that gave the foundation to what is known today as poset topology, a field that studies interactions between poset theory, topology, algebra and enumeration.

A very good example of this interaction comes from the theory of algebras and operads (an operad is an algebraic structure that encodes types of algebras). An algebra is an object that can be described in a purely algebraic manner by generators and relations. However some algebras are defined as the cohomology ring of a topological space. This topological space in turn can have a combinatorial description. For example, it can be a geometric realization of an abstract simplicial complex constructed using the totally ordered subsets of a poset. We will be able then to relate the algebraic information of the former algebraic object to combinatorial information about the poset. This is one of the core ideas that we will exploit during this course and we will develop the necessary tools to do it.

Topics:

- Basic definitions and general results about partially ordered sets
- Order complexes and poset topology
- Poset homology and cohomology
- Group actions on posets and group representations
- Shellability and edge labelings
- The Whitney homology technique
- The theory of combinatorial species
- Set monoids and set operads
- Associative algebras and algebraic operads
- Hopf monoids and combinatorial Hopf algebras

Grading: The grade in this course will be based on one presentation and the review of one paper related to the topics of the course.

Prerequisites: This course assumes familiarity with Algebra and Linear Algebra.

References

[1] François Bergeron. *Algebraic combinatorics and coinvariant spaces*. CMS Treatises in Mathematics. Canadian Mathematical Society, Ottawa, ON; A K Peters, Ltd., Wellesley, MA, 2009.

[2] Jean-Louis Loday and Bruno Vallette. *Algebraic operads*, volume 346 of *Grundlehren der Mathematischen Wissenschaften [Fundamental Principles of Mathematical Sciences]*. Springer, Heidelberg, 2012.

[3] Miguel A. Méndez. *Set operads in combinatorics and computer science*. SpringerBriefs in Mathematics. Springer, Cham, 2015.

[4] Richard P. Stanley. *Enumerative combinatorics. Vol. 2*, volume 62 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, 1999. With a foreword by Gian-Carlo Rota and appendix 1 by Sergey Fomin.

[5] Richard P. Stanley. *Enumerative combinatorics. Vol. 1*, volume 49 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, second edition, 2012.

[6] Michelle L Wachs. Poset topology: tools and applications. *arXiv preprint math/0602226*, 2006.