Exercise Session 3

Permutation Flow Combinatorics

Sage Days 130.5 – May 4 – May 8, 2026, Montréal

Permutation Flows Minicourse

This session covers the underlying combinatorial objects supporting permutation flows, in other words, the permutation flow zoo: cliques, vines, groves, and permutation flows. The more general versions of these objects, where shuffles are involved, give a triangulation of flow polytopes with more general netflow.

Manual problems

Problem 1. Construct all vines and groves for the polytopes of problem 1 of Session 2.

Problem 2. Compute the normalized volume of $\mathcal{F}_{PS_3}(1,1,1,-3)$ by using the Lidskii formula

$$\operatorname{vol} \mathcal{F}_G(\mathbf{a}) = \sum_{\mathbf{s} \rhd \mathbf{t}} \binom{m-n}{\mathbf{s}} \mathbf{a}^{\mathbf{s}} K_G(\mathbf{s}-\mathbf{t}).$$

Problem 3. Construct all permutation flow shuffles for $\mathcal{F}_{PS_3}(1,1,1,-3)$ and confirm that the number of them matches the Lidskii formula.

Coding problems

The companion notebook `coding_3.ipynb` explores vines and groves.